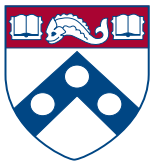

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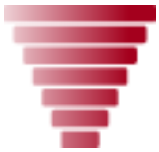

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On the The Living Arrangements of Elderly Widows *

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Abstract

Between 1970 and 1990 the share of elderly widows living alone grew by 23.2% in the U.S. (from 52.1% to 64.2%), the share living with their children decreased in a similar magnitude, while the other types of living arrangements remained stationary. In the same period there was a moderate increase in national incomes and a big increase in the income of widows. We pose a variety of models of the determination of living arrangements between widows and their children where living together provides consumption gains due to economies of scale, and it may also provide utility directly. We estimate those models using 1970's data and for some of them we obtain an excellent fit despite the fact that the data display a very nonlinear relation between living arrangements and income. We use the models to measure the contribution of income changes on changes in living arrangements. Our findings are very sharp. The simplest version of the model performs very good and it predicts that changes in the incomes of both the widow and her off-spring generates three quarters of the increase in the number of elderly widows that live alone. An extension of the basic model that takes into account the marital status of children provides slightly better estimates and imputes one half of the observed changes in living arrangements to changes in the incomes and in the marital status of the children.

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1 Introduction

Between 1970 and 1990 the share of elderly widows (which are more than one half of the women over 65) living alone grew by 23.2% in U.S. (from 52.1% to 64.2%). Those living with their children decreased in a similar magnitude, while the other types of living arrangements such as institutions or with other adults remained stationary.

In the same period there have been large changes in incomes. In 1970 the average annual income for an elderly widow was $2,162, while the average annual household income for their children was $10,556. After correcting for the mismeasurement of inflation in the CPI (Gottschalk (1997)), average incomes increased by 55.3%: those of elderly widows increased by 106.8% while those of their children increased by 52.1%. Henceforth, not only all incomes have gone up, but those of the elderly increased much more.[1]

The purpose of this paper is to investigate the role that those changes in incomes may have had in shaping the changes in the choices of living arrangements of elderly widows and of their children. In order to answer this question we pose a model where elderly widows and their children make investments to get their preferred living arrangements. The model explicitly introduces economies of scale in multi person households, differences in the risk aversion of agents, and direct preferences over the living arrangement as the trade-offs that agents face. Incomes of the mother and the children affect those trade-offs yielding different outcomes for different income groups. In fact, we pose a variety of models that differ in the specifics and that accommodate additional features such as altruism and the marital status of the children. We estimate those models using 1970's data, and for some of the models we obtain an excellent fit, replicating the joint distribution of living arrangements by income groups of elderly widows and their children. We then use the models to measure the extent to which changes in income have accounted for changes in living arrangements.

Our findings are that income changes play a central role in accounting for the increase in the number of elderly widows that live alone. A simple model that abstracts from the marital status of the children and that matches the living arrangements of 1970 very well predicts

[1] The important gains in widows' income for this period have been well documented. Hurd (1990) reports that the poverty rate of the non-elderly rose from 11.8 to 14.5 percent between 1967 and 1984. In contrast, the poverty rate among elderly widows fell from 35.1 to 19.1 percent between 1961 and 1987. Radner (1995) reports that the ratio between the income of elderly family units and the income of 65 and less years old family units increase from 0.5 in 1967 to 0.63 in 1990.

that the changes in the incomes of both the widow and her off-spring generate 74.4% of the increase in the number of elderly widows that live alone. A model that takes into account the marital status of the children and that consequently provides even better estimates for the 1970 data predicts that the changes in income and family structure among the children account for 50.4% of the observed changes in living arrangements.

When decomposing the changes, we found that the increase in the income of the widows accounts for 63.9% of the increase in the number of widows living alone. Interestingly, the model predicts that the increase in the income of the children by itself would have reduced slightly the number of widows living alone, implying a very complex and non-linear relation between income and living arrangements. From a different angle, the model imputes two thirds of the changes due to income changes to changes in relative income and one third to changes in the level of income. Finally, we found that change in income dispersion within mothers and within children only account for a very small part of the changes in living arrangements.

An important additional contribution of our work is the construction of the data. While the census provides information about family members that live in the same household, it does not connect those that live apart. The 1993 AHEAD[2] however provides this information. We make the assumption that the joint distribution of (relative) income across generations was the same in 1970 and 1990 than in 1993 (the AHEAD year), which allows us to construct pairs of mothers and children that do not live together using Census data, allowing us to estimate the models.

The living conditions of the elderly have attracted considerable attention in the literature as it is considered quite important for determining their wellbeing. There are some studies that have related the growth of income of the elderly to the increasing share of them that live alone. See McGarry and Schoeni (2001), Costa (1999), and Macunovich, Eaterlin, Schaeffer, and Crimmins (1995) for an extended survey. While Burr and Mutchler (1992) among others, argues in favor of income of the elderly being an important determinant in the change of the living arrangements, Schwartz, Danziger, and Smolensky (1984) and Börsch-Supan, Gokhale, Kotlikoff, and Morris (1992) see no role in the increase of income of the

[2]The 1993 AHEAD is the first wave of the data collection of the study of Asset and Health Dynamics among the Oldest Old (AHEAD) and it is included in the Health Retirement Study (HRS). The focus of the AHEAD is to understand the impacts and interrelationships of changes and transitions for older Americans in three areas: health, financial and family.

elderly's in the change of living arrangements patterns. All these studies have in common that they see the determination of the living arrangement as determined uniquely by the circumstances of the elderly.

Studies that consider incomes and living arrangements of the elderly in relation to characteristics of other individuals (their children mostly) are rare. Kotlikoff and Morris (1990) argues that since incomes of parents and children are correlated, the effects of parents' income on living arrangements may be capturing some influence of children's incomes. For a small and non representative sample, it finds that the probability to coreside of parents and children is negatively but not significantly correlated with the children's income. Dunn and Phillips (1998) finds that poorer, unmarried or childless siblings are more likely to live with their parents. Wolf and Soldo (1988) investigates the characteristics of children living with their parents while Ward, Logan, and Spitze (1992) underline the implicit transfers from parents to children when living together.

From a different point of view, studies that analyze the children's decision of leaving the parental household find that the higher is the children's income, the higher is the probability that they leave the parental home, and that the higher is the parents' income the higher is the probability to co-reside. See for example Whittington and Peters (1996).

McElroy (1985) and Rosenzweig and Wolpin (1993) pose and estimate structural models of the young children's decision of leaving the parental household. Rosenzweig and Wolpin (1993) analyzes the relationship between children's human capital investments and parental help (inter-household transfers when living apart and co-residence with) while McElroy (1985) shows that young children jointly choose their market work and household membership. In both of these papers, the higher is the children's income the higher is the probability that they leave their parents' home, and the higher the parents' income the more likely to co-reside.

Section 2 reports the features of the data that we are interested in: the main facts related with the joint distribution of incomes and living arrangements in 1970 and in 1990. We turn to the theory in Section 3. We pose a general model that is flexible enough to allow for different attitudes towards consumption, for different types of economies of scale and for different types of sharing arrangements. In fact, we investigate nine alternative specifications of these features. Section 4 describes how we estimate the models using 1970

data and displays the estimation results. Section 5 performs the measurement of the role of changes in income in shaping changes in living arrangements. We do this by computing the equilibria of the models that we estimated with the new values for income. Section 6 decomposes the observed changes in income by isolating the role of the changes of each group (mothers and children), by separating relative and absolute changes, and by isolating changes in income dispersion. In Section 7 we turn to explore the role of other variables in shaping the living arrangements distribution. We document the contribution of a few of those variables with special emphasis concentrating in marital status that we think is the most important. Consequently, we report the income, marital status and living arrangements distribution for both years 1970 and 1990. Section 8 poses a version of our model where the children differ in marital status and in Section 9 we estimate it. Section 10 performs the counterfactual exercise that yields the answer to our question and Section 11 concludes. The paper also includes various appendices. Appendix A describes how we select and refine the sample in both years 1970 and 1990, while in Appendix B we explain the imputation process we carry out from the 1993 AHEAD to the IPUMS database in order to complete the information we need. In Appendix C we describe the imputation process when we consider the marital status of the children, and finally, in Appendix D we report another estimation of the model with marital status.

2 The Data

To understand the determinants of the living arrangements we have to know the characteristics of elderly widows and their children when they live together and when they live apart both for 1970 and for 1990. This information is not available.[3] However in 1993 the AHEAD Survey was collected and it allows us to determine the joint distribution of income of the mothers and of their off-spring. The AHEAD collects information by interviewing directly the targeted population: individuals who are 70 years old or older, and not living in institutions. Because one of the main focus of the AHEAD is to analyze the elderly people's

[3]While the PSID allows the possibility to match different members in a family when they are living, its sample size it is too small. Note that we have to look at the subsample of elderly widows. In 1988 there was a Special Survey (The 1988 Time and Money Transfer File) that was designed to measure transfers between family members but even in this case both the sample size was too small and there were too many missing values. There were 271 non-married women older than 66, of whom 260 were living alone and the remaining 11 were living with their children.

families structures and relationships, Section D of the survey collects detailed data about the subjects' relatives, whether they are living in the same household or not. Its sample size is 17,718 individuals, of whom 8,222 are elderly people and the rest are relatives. We then turn to Census data as reported by the IPUMS (Integrated Public Use Microdata Series) samples for 1970 and 1990, a very convenient source because of its big size: 41,385 elderly widows in 1970 and 61,611 in 1990. From the IPUMS we obtain information about the living arrangements of the elderly widows and about the marginal income distributions of income both for the elderly widows and for their off-spring. We then make a central assumption: that the joint distribution of (relative) income across generations was the same in 1970 and 1990 than in 1993 the year of AHEAD. This assumption allows us to construct pairs of mothers and children, and of their respective incomes in the census samples. Appendix A describes the sample selection while Appendix B describes the imputation of income.

2.1 The Data in 1970

We distribute the elderly widows in four categories: living alone, living with children, living in institutions and living with others. The first two groups constitute 85.0% of the sample. Of these, 62.0% live alone while the remaining 38% live with their off-spring. In 1970 the average annual income for an elderly widow was \$2,162, while the average annual household income for the children was \$10,556. Income was more unequally distributed among the elderly widows than among their children in the sense that their respective Gini Indices were 0.48 and 0.38.

Our analysis of the data can be summarized with Table 1 where we have sorted mothers and their children in four equal sized income groups. We then constructed the cross product of these income groups and we report the percentage living alone within each of the sixteen groups (in boldface) and the relative size (in parenthesis) of each one of these groups. The sample size is large totaling more than 30,000 observations. Note that the size of the groups along the diagonal is larger than that of groups away from the diagonal which is an implication of the intergenerational persistence of income. The main pattern of the data is that more income tends to increase the fraction of widows that live alone. However, this pattern is not universal: for poor mothers, the higher the income of their children the less likely it is that the widows live alone. The opposite is true for widows in the top half of the

income distribution. For mothers in the second quartile of income there is an inverted U relationship. For poor children, the income of the mother does not matter much, it displays a skewed inverted U shape. For higher income children, the higher the income of the mother the higher the frequency with which they live alone. An important feature of these patters is its strong non-linearity which the estimated models will try to replicate.

Table 1: 1970 Distribution of income groups and percentage of widows living alone (bold)

	Mothers			
	0-25	25-50	50-75	75-100
0-25	**49.4** (9.1)	**50.1** (6.3)	**57.6** (5.6)	**48.8** (4.0)
Children 25-50	**56.8** (8.1)	**64.0** (6.7)	**68.4** (5.9)	**67.0** (4.3)
50-75	**31.7** (3.9)	**68.1** (7.1)	**69.2** (5.7)	**84.7** (8.3)
75-100	**23.2** (3.9)	**52.7** (4.9)	**76.7** (7.8)	**81.3** (8.4)

2.2 The Data in 1990

By 1990 things have changed quite dramatically. Incomes grew by 54.2%, especially that of the widows that went from being 20.5% of that of their children in 1970 to being 29.6% in 1990. The Gini Indices become closer to each other with values midway of those in 1970: they become 0.42 for mothers (0.48 in 1970) and 0.43 for children (0.38 in 1970).

Simultaneously there was an important change in the distribution of living arrangements.[4] The change was a shift from living with their children to living alone (the share of other arrangements maintained their 1970 share of about 15%). Excluding those other arrangements the fraction of the elderly widows living alone went from 62% in 1970 to 75.3% in 1990.

We report in Table 2 the joint distribution of income and living arrangements. The sample size is over 50,000 pairs. There has been an increase in the fraction of elderly widows that live alone in all income groups albeit not in the same proportion. For the groups with the

[4]We have increased the age of the widows that we look at to account for the increase in life expectancy. See the Appendix A for details.

poorest mothers and richest children the fraction of mothers living alone more than doubled. The increase was less dramatic for the groups consisting of mothers with higher income. The shape of the relation is very similar to that of 1970: more income implies more mothers living alone. The differences are: For the group of poorest mothers the relation between living alone and the income of their children is fluctuating between 60.4% and 77.5% (recall that in 1970 it was a decreasing relation). For the second quartile of mothers, there is an increasing relation, while in 1970 it displayed an inverted U shape. Finally, for the second quartile of children we can see a flat relation (it was increasing in 1970).

Table 2: 1990 Distribution of income groups and percentage of widows living alone (bold)

	Mothers			
	0-25	25-50	50-75	75-100
0-25	**60.7** (9.1)	**58.8** (6.3)	**63.1** (5.6)	**60.5** (4.0)
Children 25-50	**77.5** (8.1)	**73.4** (6.7)	**75.6** (5.9)	**74.0** (4.3)
50-75	**60.4** (3.9)	**82.5** (7.1)	**80.9** (5.7)	**89.0** (8.3)
75-100	**67.1** (3.9)	**80.1** (4.9)	**88.9** (7.8)	**91.5** (8.4)

Figure 1 shows compactly the properties of the data for 1970 and 1990. We can observe how similar the patterns of data are in both years 1970 and 1990 and the huge increase undergone in the groups of poor mothers especially in the ones with richest children.

3 The Model

In this section we present a model of the determination of living arrangements. In fact, we start describing a general model and then we construct various alternative specifications. The model poses large numbers of two agents pairs, a mother that we denote m, and her child, that we denote h and that differ in preferences, and in income, and they may or may not live together. The agents have preferences over consumption, denoted c, effort, denoted e, and the living arrangement, whether they live together, denoted t, or not, denoted a. In

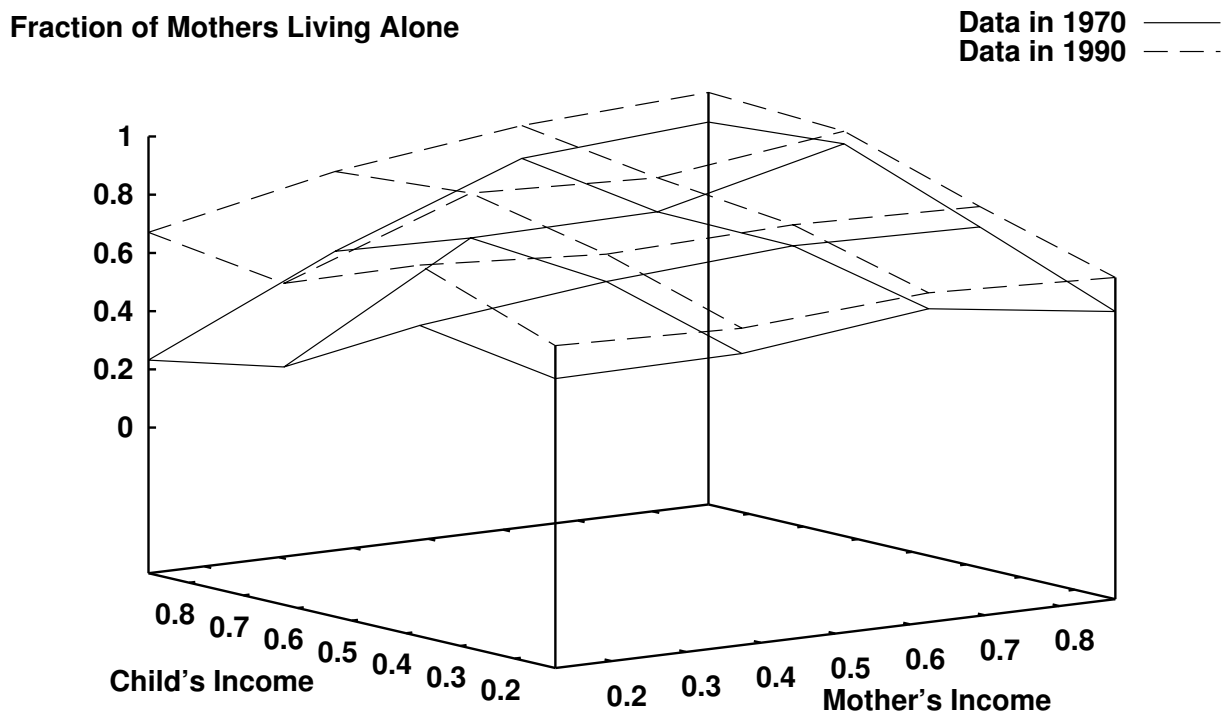

Figure 1: **Fractions of widows that live alone by income quartile of them and of their children in 1970 and 1990.**

some of the specific models that we will see below one of the agents may be altruistic towards the other. This shows up as having the other agent's utility as an argument of its own utility function.

Consumption equals income if living alone and it is the same for both agents when living together once it is normalized to account for the economies of scale within a household. The actual living arrangement is a random variable that depends on the effort posed by the agents to achieve their more desired outcome. This is perhaps slightly controversial (why do not agents live together if and only if both want to) and it requires some explanation.[5] If living together requires that both parties want to, then the estimation process is quite hard since the equilibrium outcomes are discontinuous functions of the parameters, and

[5]We follow the modeling choice of **?** and many others.

this poses problems. In addition, to this technical reason, we think that it is a reasonable assumption. For any given effort of the other agent, any agent can do what she really wants with probability one, albeit at perhaps a huge utility cost.

Let the utility of a mother that lives with her child be denoted by $u^m(c, e, t)$ and if she lives alone it is denoted by $u^m(c, e, a)$. Likewise for a child we have $u^h(c, e, t)$ and $u^h(c, e, a)$. Their respective efforts have to be determined and the natural equilibrium concept is Nash. Given their respective incomes both agents choose their effort, taking into account the other agent's choices, and how consumption depends on their living arrangement. The problem of a mother is then

$$\max_{e^m} \quad p_a(e^m, e^h) \;\; u^m[y^m, e^m, a] \;\; + \;\; [1 - p_a(e^m, e^h)] \;\; u^m[\phi_t(y^m, y^h), e^m, t] \tag{1}$$

where $p_a(e^m, e^h)$ is the probability of living alone when the effort of the mother is e^m and that of the child is e^h, and where $\phi_t(y^m, y^h)$ is the effective consumption of a mother with income y^m when living together with her child that has income y^h.

The problem of a child is then to solve

$$\max_{e^h} \quad p_a(e^m, e^h) \;\; u^h[\phi_a(y^h), e^h, a] \;\; + \;\; [1 - p_a(e^m, e^h)] \;\; u^h[\phi_t(y^m, y^h), e^h, t] \tag{2}$$

where $\phi_a(y^h)$ shows the possible economies of scale affecting the child's household when the mother lives alone.

For appropriately chosen functions u and p_a the problem is strictly concave and its solution is given by the first order conditions which are

$$0 = \frac{\partial \; p_a(e^m, e^h)}{\partial e^m} \;\; \left[u^m[y^m, e^m, a] - \; u^m[\phi_t(y^m, y^h), e^m, t]\right] \; + \; \frac{\partial u^m[y^m, e^m, a]}{\partial e^m} \;\; p_a(e^m, e^h) + \frac{\partial u^m[\phi_t(y^m, y^h), e^m, t]}{\partial e^m} \;\; [1 - p_a(e^m, e^h)] \tag{3}$$

$$0 = \frac{\partial p_a(e^m, e^h)}{\partial e^h} \left[u^h[\phi_a(y^h), e^h, a] - u^h[\phi_t(y^m, y^h), e^h, t]\right] \; + \; \frac{\partial u^h[\phi_a(y^h), e^h, a]}{\partial e^h} \;\; p_a(e^m, e^h) + \frac{\partial u^h[\phi_t(y^m, y^h), e^h, t]}{\partial e^h} \;\; [1 - p_a(e^m, e^h)] \tag{4}$$

A Nash equilibrium is just a solution to this system of equations.

Since what differs across mothers and children is their income we specify the equilibrium as a pair of functions $e^m(y^m, y^h)$ and $e^h(y^m, y^h)$ that gives the efforts of mother and child

when their respective incomes are y^m and y^h. Because there is a large number of agents, the fraction of mothers that live alone out of all pairs with income y^m and y^h is given by $p_a[e^m(y^m, y^h), e^h(y^m, y^h)]$.

A description of society with our model is just a specification of the joint distribution of income between mothers and their children and the fraction of mothers within each pair of incomes that live alone.

We next turn to describe the model in some detail. We do so by looking first at what we call the Baseline and then briefly describing how the alternatives differ.

Model 1 (Baseline): Mothers care about living arrangements; children do not. The functional form that determines how effort affects the probability of living alone is

$$p_a[[(e^m, e^h)]] = \frac{\exp\left(e^m + e^h\right)}{\exp\left(e^m + e^h\right) + \quad \rho \quad \exp - \left(e^m + e^h\right)}, \tag{5}$$

which only depends on one parameter, ρ. Note that for any pair of real numbers we obtain a probability, for example, zero effort of both parties yields a probability of living alone of $\frac{1}{1+\rho}$. Also note that since efforts have different utility costs they are not really symmetric.

With respect to the economies of scale we pose $\phi_a(y^h) = \frac{y^h}{\gamma - 0.7}$ and $\phi_t(y^m, y^h) = \frac{y^m + y^h}{\gamma}$ which also implies another parameter, γ. We are using the OECD estimations of the equivalence scales to take into account the effect of the mother in total consumption. While the first adult in the household amounts to 1, the consecutive ones are computed as 0.7. However, we used values from 0.7 to 1 and the results virtually did not change.

We specify the part of the utility function that depends on consumption as being the log of consumption minus a constant that can be either positive or negative. This yields two more parameters. Moreover, the utility that the mothers get from living with their children is η^m, which of course may be negative. This is a fifth parameter.

Effort generates a direct disutility, and we pose it as $-\alpha^m \ (e^m)^2$, and $-\alpha^h \ \left(e^h\right)^2$, where the α's are positive parameters. Note that this function is convex, implying that the more effort an agent puts, the higher the marginal disutility that it poses. This implies two more parameters, yielding a total of seven.

The utility function of a mother is then given by

$$u^m = -\alpha^m \left(e^m\right)^2 + p_a\left(e^m, e^h\right) \log\left(y^m - \overline{c}^m\right) + \left[1 - p_a\left(e^m, e^h\right)\right] \left[\log\left(\frac{y^m + y^h}{\gamma} - \overline{c}^m\right) + \eta^m\right] \tag{6}$$

while that of the child is

$$u^h = -\alpha^h \left(e^h\right)^2 + p_a\left(e^m, e^h\right) \log\left(\frac{y^h}{\gamma - 0.7} - \overline{c}^h\right) + \left[1 - p_a\left(e^m, e^h\right)\right] \log\left(\frac{y^m + y^h}{\gamma} - \overline{c}^h\right) \tag{7}$$

Model 2: Children care about living arrangements; mothers do not. This model differs from the previous one in the fact that the living arrangement enters in the utility of the child but not of the mother. The term η^m disappears from equation (6) and η^h is added to the child's utility in equation (7). The model also has seven parameters.

Model 3: Both care about living arrangements. This model is the union of the previous two ones and it has eight parameters.

Model 4: Both care about living arrangements; mothers also care about their children. This model is like the previous one with an added term in the utility of the mother that has the utility of her child multiplied by a parameter, φ^m, that measures the strength of the altruism. This model has nine parameters.

Model 5: Both care about living arrangements; children also care about their mothers. This is the symmetric case to the previous one. Both agents care about the living arrangement and the child's utility function has a term with the utility of the mother weighted by φ^h. This model also has nine parameters.

Model 6: Mothers care about living arrangements; children also care about their mothers. This model is like the previous one, except the child does not care about the living arrangement. It is like the Baseline with the added altruistic term. This model has eight parameters.

Model 7: Mothers care about living arrangements; both agents are altruistic. In this model mothers care about the living arrangement while their children do not and both mothers and children are altruistic. This model has nine parameters.

Model 8: Baseline with a new effort function. This model differs from the Baseline in the effort function. The function that we choose is also a one parameter function that is centered around one half (zero effort of all parties yields a 0.5 probability of living alone). In this case we give different abilities to affect the odds of living alone to both agents. The new effort function is

$$p_a(e^m, e^h) = \frac{\exp\left(e^m + \rho_1 e^h\right)}{\exp\left(e^m + \rho_1 e^h\right) + \exp - \left(e^m + \rho_1 e^h\right)} \tag{8}$$

Model 9: Baseline with a two parameter effort function. Our last model is another variation of the Baseline. The twist is now that the effort function is a two parameter function that allows for centering at $\frac{1}{1+\rho}$ and for differential effects of the mother and her child. The effort function is

$$p_a(e^m, e^h) = \frac{\exp\left(e^m + \rho_1 e^h\right)}{\exp\left(e^m + \rho_1 e^h\right) + \rho \exp - \left(e^m + \rho_1 e^h\right)} \tag{9}$$

4 Estimation

The next step is to obtain parameters for our various models from the 1970 data. The way we proceed is to construct various pairs of mothers and children with incomes that match the data. We start by sorting mothers and their children into four equal sized income levels. Then we construct the product pairs of mothers and children according to these criteria obtaining sixteen cells. Note that we are using between seven and nine parameters to get sixteen targets.

4.1 Estimation Procedure

The estimation procedure that we use is a minimization of the weighted sum of the squares of the differences between the fraction of single mothers generated by the model and the

data within each of the sixteen income groups subject to the requirement that they match the aggregate fraction of single mothers in the data. We have used as weights for all income groups the actual relative size of the groups. Because of the intergenerational persistence of income the groups in the diagonal of the Table are generally larger. We obtained very similar estimates using equal weights across groups.

4.2 Estimation Results

We now turn to report the estimates of the nine models using 1970 data. We report how the living arrangements in the models compare with the data, and also a measure of accuracy which is essentially the fraction of the variance of living arrangements accounted for by the model. Formally,

$$\text{Accuracy} = 1 - \frac{\sum_{i,j}^{4} \left(A_{i,j} - p_a(e_i^m, e_j^h)\right)^2 \; P_{i,j}}{\sum_{i,j}^{4} \left(A_{i,j} - 0.62\right)^2 \; P_{i,j}} \tag{10}$$

where $P_{i,j}$ is the proportion of mothers of income type j with children of income type i, $A_{i,j}$ is the share of elderly widows of type $\{i, j\}$ who live alone in the data, and $p_a(e_i^m, e_j^h)$ is the model's counterpart.

Table 3: Predictions of Model 1 for 1970
Percentage of mothers living alone

Accur.		Mother							
0.8854		0-25		25-50		50-75		75-100	
	0-25	47.3	**(49.4)**	55.6	**(50.1)**	58.1	**(57.6)**	46.3	**(48.8)**
Child	25-50	48.9	**(56.8)**	67.3	**(64.0)**	72.6	**(68.4)**	78.8	**(67.0)**
	50-75	40.4	**(31.7)**	65.9	**(68.1)**	72.6	**(69.2)**	80.6	**(84.7)**
	75-100	23.8	**(23.2)**	56.9	**(52.7)**	68.6	**(76.7)**	80.9	**(81.3)**

Model 1 (The Baseline) Table 3 shows the predictions of the Baseline as well as its accuracy measure. To better assess the model, the Table also includes the corresponding values of the data. We see that the model replicates the features of the data despite their strong non-linearities. Recall that while in general more income implies a higher proportion of individuals living alone, this is not the case either for the poorest mothers for whom the income of the children increases the fraction living together, nor for the poorest children for

whom the behavior is more of an inverted U shape. The model traces the data very well with a relatively small number of parameters producing increases of different steepness in different directions. To give a graphical sense of the accuracy of the model, Figure 2 shows the distribution of living arrangements both in the Baseline model and in the data.

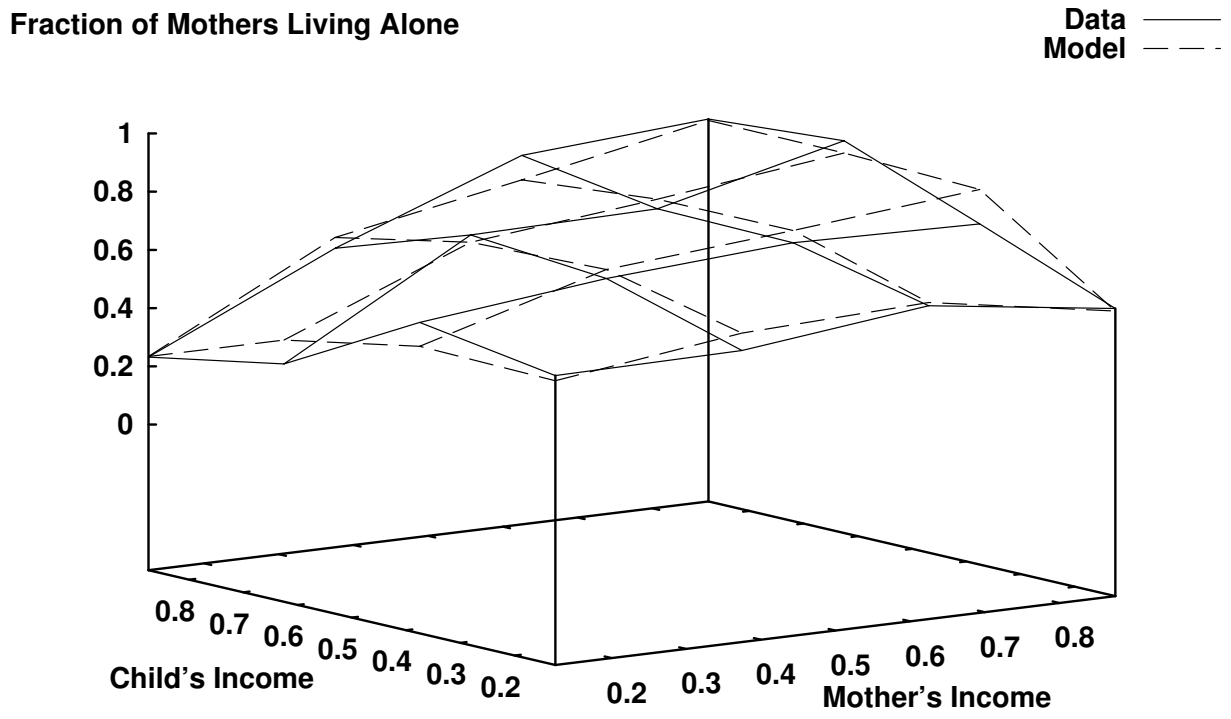

Figure 2: **Fraction of mothers living alone in the Baseline model and in the 1970 data.**

The first column of Table 4 shows the values of the parameters yielded by the estimation procedure. We see that the minimal consumption parameter of the mothers is negative indicating that they are not too risk averse especially at low levels of consumption, while the opposite holds for the children. We also see that the parameter estimates imply that with zero efforts, the probability of living alone is 0.2, indicating that typically agents place costly effort to live alone. The effort seems to be more costly to the mother, and she prefers to live alone. Finally, the estimates of the economies of scale in the household indicate that

Table 4: Parameter Estimates

	Model								
	1	2	3	4	5	6	7	8	9
$\overline{c}^m$	-725.36	-559.68	-725.36	-915.11	-915.02	-920.84	-920.84	-1669.75	-725.36
$\overline{c}^h$	22.49	64.19	20.14	27.24	20.71	26.46	26.46	-4.25	20.01
ρ	3.97	2.91	3.94	6.75	4.31	4.27	4.27	–	3.94
ρ_1	–	–	–	–	–	–	–	4.38	-0.00
α^m	0.14	0.18	0.15	0.14	190.97	189.77	189.77	0.08	0.15
α^h	0.11	0.21	0.11	0.21	0.11	0.11	0.11	1.33	0.11
η^m	-0.20	–	-0.20	0.70	-0.23	-0.23	-0.23	0.06	-0.20
η^h	–	-0.18	0.00	-0.90	0.00	–	–	–	–
φ^m	–	–	–	0.51	–	–	0.0	–	–
φ^h	–	–	–	–	0.85	0.85	0.85	–	–
γ	33.90	32.17	32.95	31.20	31.22	31.22	31.22	26.22	33.06
Accur.	0.8854	0.8751	0.8854	0.8867	0.8854	0.8854	0.8854	0.8468	0.8854

the marginal contribution of the mother to a household that includes her and her child is very small.

We now turn to briefly describe the results of the alternative models. Table 5 shows the predictions of most of these models while Table 4 shows the parameter estimates.

Model 2 (Children care, mothers do not): This model yields similar but slightly worse (lower accuracy) results than the Baseline. The parameter estimates change a little especially the minimal consumption of the child that is now larger. The estimates also show that the child would rather live alone than with the mother.

Model 3 (Both agents care): This model is richer than the previous two in the sense that it has one more parameter and, hence, more possibilities of matching the data. However, there is no increase in accuracy with respect to the Baseline (in fact the estimate of the extra parameter is zero, the value that is implicitly assumed in the Baseline). We conclude that the simultaneous inclusion of the mother and the child caring directly for the living arrangement is not a useful modeling strategy.

Model 4 (Both agents care and the mother is altruistic): In this model, there are nine parameters, and it has the Baseline as a special case, but the gains in accuracy are small. The estimates change the attitude of the mother with respect to living together, now she is slightly in favor of it, but the child is not. However, given that the mother is altruistic with respect to her child, the net effect is close to zero.

Table 5: Predictions of All Models for the % of Mothers Living alone

		Mothers				Mothers			
		0-25	25-50	50-75	75-100	0-25	25-50	50-75	75-100
		Model 2, Acc. 0.8751				**Model 3, Acc. 0.8854**			
	0-25	45.0	51.5	55.4	49.4	46.9	55.5	58.1	47.1
Child	25-50	49.8	68.5	73.9	80.4	47.6	66.4	71.9	72.2
	50-75	40.3	66.0	72.9	81.1	39.1	64.8	71.8	80.0
	75-100	24.8	55.0	67.4	80.3	23.3	55.3	67.5	80.3
		Model 4, Acc. 0.8867				**Model 5, Acc. 0.8854**			
	0-25	46.7	53.9	56.5	48.4	45.9	53.6	56.5	48.5
Child	25-50	49.0	65.9	71.2	77.9	48.1	66.4	72.1	79.0
	50-75	40.3	64.6	71.2	79.7	39.7	65.0	71.1	80.7
	75-100	20.7	55.9	67.3	79.9	23.1	55.6	67.9	81.0
		Model 6, Acc. 0.8854				**Model 8, Acc. 0.8468**			
	0-25	45.9	53.5	56.5	48.5	47.0	52.4	55.0	46.1
Child	25-50	48.1	66.3	72.0	79.0	49.3	69.0	75.9	83.8
	50-75	39.7	64.9	71.8	80.7	42.1	67.9	76.5	85.8
	75-100	23.2	55.6	67.9	81.0	28.3	56.0	72.2	86.6

Model 5 (Both agents care and the child is altruistic): This is another nine parameter model like the previous one, and its accuracy is lower than the previous one (it is about the same as the Baseline despite having more parameters). The altruism parameter is positive and the effort is very costly for the mother. We think that this model is not a good one.

Estimation of Model 6 (Mother cares about arrangement, child is altruistic): This is an eight parameter model, yet it gives almost the same predictions as Model 5. The reason for this is that η^h, the additional parameter in Model 5 had an estimate of 0.0, the assumed value in Model 6.

Estimation of Model 7 (Mother cares about arrangement, both agents are altruistic): This is identical to Model 6, since the point estimate of the extra parameter used in this model, φ^m is zero.

Estimation of Model 8 (Baseline with alternative effort function): This model is a version of the Baseline where the effort function discriminates between the ability of the mother and of the child in affecting outcomes. This model does not improve over the Baseline, in fact, it does quite worse. Moreover, the estimates change. For example, now the minimum consumption of the child is a lot smaller (even negative). We do not think that this model provides a good estimation.

Estimation of Model 9 (Baseline with two parameter effort function): This model shows no improvement over the Baseline despite the Baseline being nested in it. The estimate of the additional parameter (ρ_1) is zero which is the implicit value in the Baseline.

To summarize the findings of this section, the Baseline model economy does a very good job in matching the data. Moreover all the variations that we have examined either provided a worse fit or required additional parameters for an increase of the measures of accuracy of 4%. Model 4, where both mother and child care about the arrangement and the mother is also altruistic towards her child, provides more accuracy than the Baseline but is a nine parameter model. We conclude that the Baseline model is good enough to study what are the implications of the changes in incomes up to 1990 for the changes in living arrangements.

5 The model's predictions for 1990

We now turn to use the model to assess the role of changes in income in accounting for the changes in living arrangements that have happened between 1970 and 1990. Note that we no

longer try to match the data: we use the model to measure the extent to which the changes in income that occurred in that period are behind the changes in living arrangements.

To this end we construct a measure of the change of the living arrangements between 1970 and 1990. We then compute the equilibrium when the incomes are those of 1990 and the parameter values are those that we estimated using 1970 data. Next, we compute a measure of the error between the predictions for 1990 of our model and the actual 1990 data. We say that our model accounts for the fraction of the change in living arrangements that results from the difference between 1 and the ratio of the prediction error of our model and the actual allocational change. Formally:

$$\text{Model accounts for} = 1 - \frac{\sum_{i,j}^{4} \left(A_{i,j}^{90} - p_a^{90}(e_i^m, e_j^h)\right)^2 \; P_{i,j}}{\sum_{i,j}^{4} \left(A_{i,j}^{90} - A_{i,j}^{70}\right)^2 \; P_{i,j}} \tag{11}$$

where $P_{i,j}$ is defined as before, $A_{i,j}^t$ is the fraction of pairs of type $\{i,j\}$ who lived alone in year $t \in \{70, 90\}$ in the data, $p_a^{90}(e_i^m, e_j^h)$ is the equivalent fraction of elderly widows living alone predicted by the model when using the parameter estimates from the 1970 data and the actual incomes of 1990.

An issue that turns out to matter for calculating the predictions of our model is the choice of price deflator to compare incomes between 1970 and 1990. While the CPI is the most popular price index, there is a relative consensus among economists that it overestimates inflation[6] so we have corrected this bias.[7] The CPI Advisory Commission calculation was 1.5 annual percentage points for the total bias in the CPI for the last decade, with a range extending from 1.0 to 2.7 percentage points per year. While the unadjusted CPI index states that \$1 in 1970 is \$3.37 in 1990, the recommendation of the Advisory Commission implies that \$1 in 1970 equates to \$2.55 in 1990. We used the adjusted CPI.

5.1 Predictions for 1990 of Model 1 (Baseline)

Table 6 reports the predictions of Model 1 when we use the fact that one 1970 dollar equates to 2.55 1990 dollars. First note that while in the data in 1990 75.3% of the widows live alone, our model predicts 71.9%. The value in 1970 in the data was 62.0%. So our model

[6] According to Gottschalk (1997), the CPI fails to capture improvements in goods quality and the ability of consumers to substitute away from goods which experience a sudden increase in prices.

[7] The same procedure was followed in Regalia and Ríos-Rull (1998).

predicts 74.4% of the increase. Using the accounting statistic defined above to measure the

Table 6: Predictions of Model 1 for 1990
Percentage of mothers living alone. Total alone predicted 71.9%

Error:		Mother							
0.00584		0-25		25-50		50-75		75-100	
	0-25	61.5	**(60.7)**	64.7	**(58.8)**	64.6	**(63.1)**	52.4	**(60.5)**
Child	25-50	70.1	**(77.5)**	76.4	**(73.4)**	79.0	**(75.6)**	81.0	**(74.0)**
	50-75	65.7	**(60.4)**	75.9	**(83.5)**	79.6	**(80.9)**	83.3	**(89.0)**
	75-100	46.1	**(67.1)**	70.0	**(80.1)**	76.9	**(88.0)**	83.4	**(91.5)**

contribution of income changes on the changes in living arrangements, we say that model accounts for 77.3% of the change in the number of widows living alone, a very similar number to the cruder measure we used before. Figure 3 depicts the prediction of the model for the 1990 data.

5.2 Predictions for 1990 of Model 4 (Both agents care and the mother is altruistic)

Recall that Model 4 was the one with the best fit to the 1970 data, even if it did so with nine parameters instead of seven. Table 7 shows the model's predictions for the living arrangements with the incomes of 1990. The model predicts 73.7% of the increase in the data. The value of the accounting statistic is 76.9%.

Table 7: Predictions of Model 4 for 1990
Percentage of mothers living alone. Total alone predicted 71.8%

Error:		Mother							
0.00594		0-25		25-50		50-75		75-100	
	0-25	61.0	**(60.7)**	64.1	**(58.8)**	65.2	**(63.1)**	58.5	**(60.5)**
Child	25-50	69.4	**(77.5)**	75.6	**(73.4)**	78.5	**(75.6)**	81.0	**(74.0)**
	50-75	65.4	**(60.4)**	75.1	**(83.5)**	79.0	**(80.9)**	83.0	**(89.0)**
	75-100	46.2	**(67.1)**	69.4	**(80.1)**	76.3	**(88.0)**	83.0	**(91.5)**

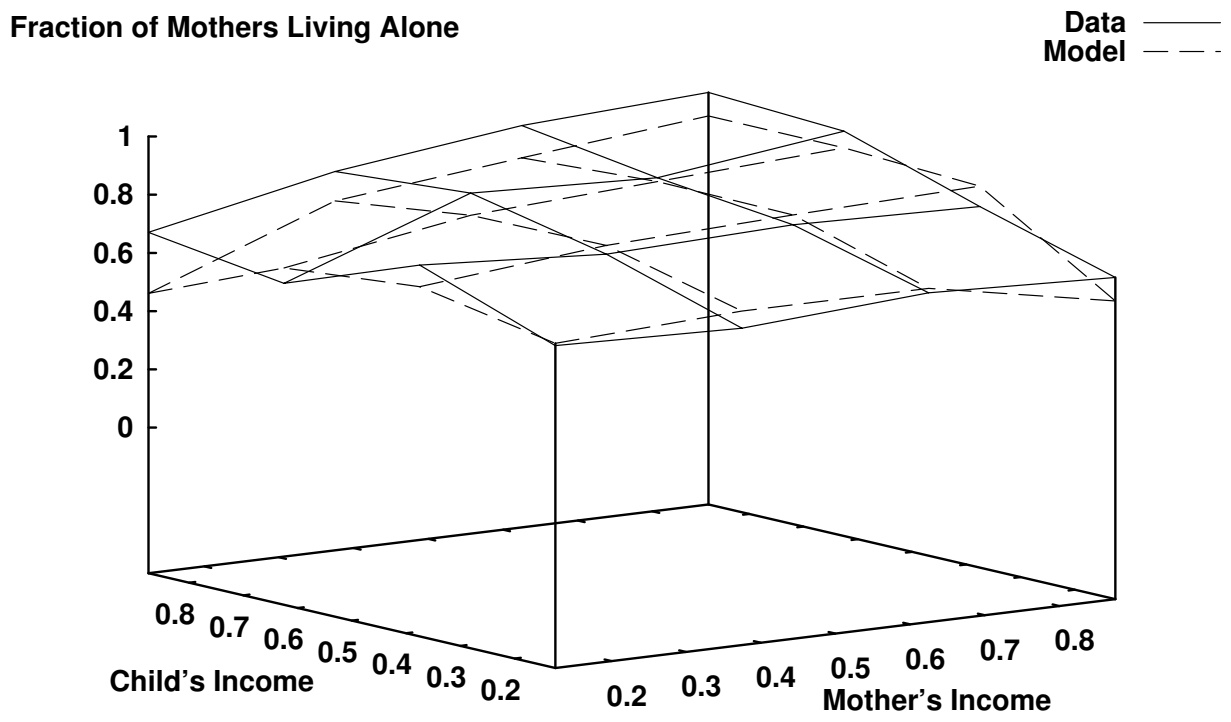

Figure 3: **Fraction of mothers living alone in the Baseline model with 1990 income and in the 1990 data.**

6 Decomposition of the change

We now turn to analyze what is the role of the different features of the changes in incomes in shaping the changes between 1970 and 1990. We decompose the changes in income in three different ways. We start by looking separately at a change in the incomes of mothers and of children in Section 6.1, we then turn to analyze the change in the level of incomes and in relative incomes in Section 6.2 and finally, in Section 6.3 we study separately the changes in the dispersions and in the averages of incomes.

Table 8: Income changes by agent type

Mothers income change, children do not. Total alone: 70.5%

Error:		Mothers							
0.00796		0-25		25-50		50-75		75-100	
	0-25	57.5	**(60.7)**	57.6	**(58.8)**	55.3	**(63.1)**	27.1	**(60.5)**
Child	25-50	70.2	**(77.5)**	75.8	**(73.4)**	78.1	**(75.6)**	79.2	**(74.0)**
	50-75	68.7	**(60.4)**	76.5	**(83.5)**	79.5	**(80.9)**	82.1	**(89.0)**
	75-100	61.6	**(67.1)**	74.7	**(80.1)**	79.1	**(88.0)**	83.6	**(91.5)**

Mothers income do not change, children do. Total alone: 60.4%

Error:		Mother							
0.04420		0-25		25-50		50-75		75-100	
	0-25	49.3	**(60.7)**	60.2	**(58.8)**	63.8	**(63.1)**	63.2	**(60.5)**
Child	25-50	45.4	**(77.5)**	66.8	**(73.4)**	72.9	**(75.6)**	79.9	**(74.0)**
	50-75	30.4	**(60.4)**	62.0	**(83.5)**	70.9	**(80.9)**	82.0	**(89.0)**
	75-100	16.0	**(67.1)**	38.4	**(80.1)**	59.9	**(88.0)**	79.3	**(91.5)**

6.1 Separate income changes by type of agent

The first panel of Table 8 reports the results of setting mothers' incomes as in 1990 and children's incomes as in 1970. We see that this change of income alone predicts a large increase in the fraction of mothers living alone, 63.9% of the total change in the data (and 85.9% of the total predicted increase by all income changes in the model). The second panel of Table 8) shows that the model predicts that 1.6% of the widows that in 1970 were living alone would have now been living with their children when the mothers' incomes are as in 1970, and the children's incomes are as in 1990, a sharp contrast with the previous case.

Our main finding is that the increase in the income of the widows is the most responsible factor for the increase in the fraction of widows living alone. Another important finding is that while the increase in the income of children by itself reduces the fraction of widows living alone, this is not the case when in combination with an increase in the widows' income. In fact the marginal contribution of an increase in the children's income after an increase in the widows' income is to further increase the fraction of mothers living alone. This is due to

the important non-linearities that are present both in the model and in the data.

6.2 Changes in relative income

To look at the effects of relative rather than absolute income changes we pose a change of the income of the mother so that it achieves the same relative income of 1990 but without the income of the children changing. The results are in the first panel of Table 9. We see an enormous increase of the fraction living alone. This increase is 45.1% of the total increase in the data (and 60.6% of the total increase predicted by the model). This shows that an increase in the relative income of the mothers is very important. Alternatively, we look at the effects of absolute but not relative changes of income by looking at the increase in income for all parties but only in the same proportion as the increase of the income of the children. Now the increase is 27.1% of the increase in the data (which is 36.4% of the total increase predicted by the model). The second panel of Table 9 shows the results.

Table 9: Relative and absolute changes in income

Children's income as 1970, mothers' relative income as 1990. Total alone: 68.0%

Error:		Mothers							
0.01110		0-25		25-50		50-75		75-100	
	0-25	56.9	**(60.7)**	57.8	**(58.8)**	56.8	**(63.1)**	33.7	**(60.5)**
Child	25-50	67.0	**(77.5)**	74.0	**(73.4)**	77.0	**(75.6)**	79.4	**(74.0)**
	50-75	61.5	**(60.4)**	72.8	**(83.5)**	77.2	**(80.9)**	81.6	**(89.0)**
	75-100	43.0	**(67.1)**	66.4	**(80.1)**	74.2	**(88.0)**	81.8	**(91.5)**

Children's income as 1990, mothers' relative income as 1970. Total alone: 65.6%

Error:		Mother							
0.02174		0-25		25-50		50-75		75-100	
	0-25	51.0	**(60.7)**	61.3	**(58.8)**	64.5	**(63.1)**	61.1	**(60.5)**
Child	25-50	52.2	**(77.5)**	70.2	**(73.4)**	75.1	**(75.6)**	80.5	**(74.0)**
	50-75	43.1	**(60.4)**	69.7	**(83.5)**	75.7	**(80.9)**	82.5	**(89.0)**
	75-100	22.1	**(67.1)**	61.0	**(80.1)**	72.0	**(88.0)**	82.7	**(91.5)**

With respect to the decomposition of the change in income between absolute and relative

changes, we see that the change in relative income accounts for almost two thirds on the predicted increase while the change in absolute income accounts for one third.

6.3 Changes in income dispersion

The top panel of Table 10 reports the predictions of the model if the averages of incomes are set to their 1970 values and the coefficients of variation are set to their 1990 values. The alternative exercise (1990 averages and 1970 dispersions) is reported in the bottom panel of Table 10. Note that even though there were relatively large changes in the dispersion of incomes (the coefficient of variation for the income of mothers decreased from 0.706 to 0.536 while it increased for the income of children income from 0.394 to 0.538), the effects of these changes as described in the top panel of Table 10 are minuscule relative to those implied by the changes in averages reported in the bottom panel.

Table 10: Changes in the dispersion of incomes

1970's incomes, 1990's coefficients of variation Total alone: 60.3%

Error:		Mothers							
0.03111		0-25		25-50		50-75		75-100	
	0-25	40.2	**(60.7)**	41.6	**(58.8)**	41.4	**(63.1)**	27.7	**(60.5)**
Child	25-50	58.2	**(77.5)**	68.0	**(73.4)**	72.7	**(75.6)**	77.3	**(74.0)**
	50-75	51.6	**(60.4)**	67.3	**(83.5)**	73.5	**(80.9)**	80.2	**(89.0)**
	75-100	29.2	**(67.1)**	57.1	**(80.1)**	69.1	**(88.0)**	80.1	**(91.5)**

1970's coefficients of variation, 1990's incomes. Total alone: 71.8%

Error:		Mother							
0.01065		0-25		25-50		50-75		75-100	
	0-25	63.3	**(60.7)**	70.1	**(58.8)**	71.5	**(63.1)**	63.5	**(60.5)**
Child	25-50	63.2	**(77.5)**	75.9	**(73.4)**	79.0	**(75.6)**	82.0	**(74.0)**
	50-75	56.5	**(60.4)**	75.0	**(83.5)**	79.0	**(80.9)**	83.5	**(89.0)**
	75-100	33.4	**(67.1)**	69.8	**(80.1)**	76.6	**(88.0)**	83.9	**(91.5)**

7 Other characteristics of individuals: marital status

In previous sections we constructed and estimated a set of models where mothers and children decide their living arrangements taking into account only their incomes. Now we ask whether there exist other variables that are relevant in shaping their decisions and whether these variables could improve the performance of the models.

7.1 Data and facts

Table 11 summarizes our findings about how various characteristics of the pairs are related to living arrangements.

Table 11: Percentage of mothers living alone per selected characteristics of their children

	0 Child	1 Child	2 Child.	3 Child.	More than 3
Total # of children		69.0	78.9	77.8	62.4
Total # of daughters	73.4	71.5	76.8	74.0	55.0
Total # of sons	73.5	75.7	71.7	67.3	56.5
Total # of married ch.	47.0	69.9	81.5	77.2	75.0
Total # of single ch.	91.5	62.6	53.7	50.0	36.4

Number of children: We see that there is a relation between the number of children and the fraction of elderly widows living alone. In fact, from one to two children the fraction of mothers living alone goes up. From then on this fraction goes down. The interaction is strongly non-linear.

Sex of the children: With respect to the sex of the children we see that those mothers with only sons or daughters are equally likely to live alone, and the numbers for higher number of children do not differ by much. While males encompass 49.4% of the children, they are 50.4% of those that live with their mother. To study the effect of the sex composition, we sorted into four groups the mothers on the basis of the proportion of daughters among their children. We found that there was not a big difference among those that have relatively high or low proportion of daughters (the fractions living alone range from 70.8% to 73.7%).

Marital status of the children: In the absence of single children more than 90% of the widows live alone while without married children less than 50% of widows live alone. Increasing the number of single children decreases the proportion of widows living alone while the opposite is true for married children (note also that while married children constitute 68.1% of all children, those that live with their mothers are 79.3% single). We also sorted the mothers by the proportion of married children. The percentages living alone for quartiles are 45.3%, 56.1%, 61.6% and 87.7% respectively. We conclude that marital status of the children seems to be closely related to the determination of the living arrangement.

Age of the children: Another feature that seems to matter is the age of the children: The mother stays with the youngest child 42% of the time while with the oldest 23%. We report in Table 12 the relation between the age of the average child and the living arrangement. Clearly, the younger the age of the average child the less likely that a mother lives alone. While this suggests to some extent that it may be important to model the relation between

Table 12: Living Arrangements by average age of the children in %

Age of Average child	Percentage living alone
< 40	60.7
40-45	67.8
45-50	72.1
50-55	77.2
$55 >$	76.6

the living arrangements of mothers and their children dynamically (mothers live with their single children then move alone, then move back in with either a single or a married child), we also observe that children are pretty well concentrated in the middle of the age ranges (75% of mothers have children with an average age between 40 and 55 years old). In any case, we leave for the future modeling dynamic considerations of the determination of living arrangements.

To summarize, marital status and, to a lesser extent, the age of children are relevant variables in shaping the living arrangements. The number of children may be relevant but it is so in such non obvious patterns that it defies simple modeling attempts. It also seems that the sex of children does not matter too much. We next incorporate marital status in a

version of our model.

7.2 Living Arrangements by Marital Status and Income Levels in 1970

As in Section 2.1 we assume that the joint distribution of relative incomes across generations by marital status was the same for 1970, 1990 and 1993 to get the right information about mothers and children that do not live together. Appendix C provides a detailed explanation about the construction of mothers-children pairs when we consider the marital status and the income jointly.

Within our 1970 sample, 57.3% of mothers are paired with a married child while 42.7% are paired with a single child. Among those paired with a married child, 70.7% live alone, a much larger number than the 50.2% that live alone when paired with a single child. Table 13 shows the living arrangements by marital status of the child and by income groups of mothers and children. For the most part, we see that for all the mothers' income groups the effect of the income of their children goes in different directions when the child is single more than when the child is married. For married children a high income increases the chance that their mothers live with them. For single children, on the whole a high income reduces the fraction of those that live together. With respect to the income of the mothers, note that more income implies living alone more often when paired with a married children while this relation is not as strong for mothers paired with a single children. We also see a couple of observations that look like outliers. The group 3,3 for single children has a much lower value than what casual extrapolation of neighboring groups indicates. Also the group 2,1 of single children looks too low. This may affect the ability of any model to match the data and we discuss it below.

Table 13: Living arrangements in 1970 by income quartiles and marital status

		Mothers							
		With Married Child				With Single Child			
		0-25	25-50	50-75	75-100	0-25	25-50	50-75	75-100
Children	0-25	65.4	76.1	87.6	88.5	34.4	42.9	41.0	39.1
	25-50	71.7	76.3	80.6	83.8	27.8	52.2	52.0	50.0
	60-75	35.3	68.5	74.5	89.2	38.5	61.5	45.3	75.7
	75-100	15.2	44.3	72.5	80.8	58.3	57.7	84.1	81.2

7.3 Living Arrangements by Marital Status and Income Levels in 1990

By 1990 there was a sharp increase in the fraction of elderly widows living alone both among those with married children and those with single children. Among those with a married child it went from 70.7% to 85.5%, an increase of almost 21%. For singles the share living alone went from 50.2% to 66.3%, an increase of 32%. The properties of the data are reported in detail in Table 14 where we can observe that for the groups with the poorest mothers and richest children the fraction of mothers living alone more than doubled. In fact, the group for which it has increased the most is for poor mothers with relatively rich married children. Note also that the value of the cells that looked "atypical" in 1970 no longer do so in 1990.

Table 14: Living arrangements in 1990 by income quartiles and marital status

		Mothers							
		With Married Child				With Single Child			
		0-25	25-50	50-75	75-100	0-25	25-50	50-75	75-100
Children	0-25	84.6	88.7	94.3	96.5	49.1	51.3	51.6	53.7
	25-50	86.2	85.6	89.4	91.8	62.4	66.9	66.7	65.2
	60-75	57.6	85.5	87.5	94.6	69.0	78.7	67.0	82.5
	75-100	62.5	77.2	86.7	91.3	81.7	77.3	91.7	91.6

8 The Model with marital status

We now present a model of the determination of living arrangements that takes into account the marital status of the children. The structure and notation of this model is the same as that of the models in Section 3: Individuals consume their incomes and they have to choose the optimal effort to determine the probability of living alone. We explored many alternative specifications by restricting some but not all parameters to be equal across married and single children pairs. The specification that we report is the one that works best. Marital status affects the probability of living alone function, the utility functions, and the budget constraints.

Probability of living alone: Let the marital status of the children be denoted as z. Children can be married, n, or single, s. Then, the probability of living alone is

$$p_a^z[[(e^m, e^z)]] = \frac{\exp(e^m + e^z)}{\exp(e^m + e^z) + \rho^z \exp - (e^m + e^z)} \tag{12}$$

where ρ^z reflects the possibility that the marital status of the children could affect it. If the mother and the child make zero effort, then the probability to be alone is $\frac{1}{1+\rho^n}$ for married children, and $\frac{1}{1+\rho^s}$ for single children.

Utility function: The utility function of the mother is:

$$u^m = -\alpha^{zm}(e^m)^2 + p_a^z(e^m, e^z)\log(c^{m,a} - \overline{c}^m) + [1 - p_a^z(e^m, e^z)]\left[\log\left(c^{m,t} - \overline{c}^m\right) + \eta^z\right] \tag{13}$$

In this model, both the cost of making effort, $-\alpha^{zm}(e^m)^2$, and the direct utility that the mother gets for living with her child, η^z, depend on the child's marital status, while the utility derived from consumption does not. The utility function of the child is:

$$u^h = -\alpha^{zh}(e^h)^2 + p_a^z\left(e^m, e^h\right)\log\left(c^{h,a}\right) + \left[1 - p_a^z\left(e^m, e^h\right)\right]\log\left(c^{h,t}\right) \tag{14}$$

Like for the mother, the effort cost, $-\alpha^{zh}(e^h)^2$, depends on the child's marital status, while the utility from consumption does not.

Budget constraints: For this model we use a particular form of increasing returns to scale in local consumption that is non linear (in contrast with standard specifications such as that of OECD weights). These budget constraints are if alone $c^{m,a} = y^m$ for the mother and $c^{h,a} = \frac{y^h}{\gamma}$ for the child (we assume that $\gamma = 1$ in the case of single children). While if together, we pose that total private consumption is $c^T = \chi^z\left(y^m + y^h\right)^{\theta^z}$ where the share that goes to the mother is $c^{m,t} = c^T\lambda^z$ and the rest, that goes to the child's family, is normalized by size yielding $c^{h,t} = \frac{c^T}{\gamma}(1 - \lambda^z)$.

The equilibrium is as before: both agents determine their efforts given their respective incomes, taking into account the other agent's choices, and how consumption depends on their living arrangements.

With all these ingredients the model has sixteen parameters: the parameter that accompanies the mother's consumption in the log utility function, $\overline{c}^m$; the mother's direct utility

from living together which depends on the child's marital status, $\eta^z,\ z = n, s$; four parameters from the effort cost functions, α^{zm} and α^{zh}, $z = n, s$; two parameters in the probability of living alone function, $\rho^z,\ z = n, s$; economies of scale for children living alone, γ; four parameters to state the economies of scale when agents live together, χ^z and θ^z, $z = n, s$; and two consumption sharing parameters, in this case, $\lambda^z,\ z = n, s$.

9 Estimation

The estimation procedure is as before, except that we now distinguish by marital status, that is, a minimization of the weighted sum of the squares of the differences between the fraction of mothers living alone generated by the model and the data within each income groups, subject to the requirement that they match the aggregate fraction of mothers living alone in the data.

Recall that when we analyze the living arrangements distribution at Section 7.2, we identified several observations as possible outliers in the sample with single children. The observations that seemed inconsistent with the rest of the patterns are cells 2,1 and 3,3. As a result, the estimation suffers and the accuracy of the model in the sample with singles was very small compared to the one in the married case. We then decided to use interpolated values for the estimation targets. More precisely, we replaced the values of cells 2,1 and 3,3 that are 27.8 and 45.3 in the data with 39.0 and 68.0 respectively. The aggregate fraction of elderly mothers living alone is now 62.6%.

9.1 Estimation Results

The model's predictions for 1970 (when using in the estimation the interpolated values) and the accuracy defined as in Section 4 are in Table 15. We observe that the model replicates the key facts: that the richer the mother the more likely they live alone for both married and single children. That the richer the child the more likely they live together if a married child and the more likely they live apart if a single child, and that for the single poorest children there is a flat relation between the mother's income and the living arrangement. We think that the performance of the model is very good.

Table 15: Predictions of the Model with marital status for 1970
Percentage of mothers living alone

Accur. **0.9314**		Mothers							
		With Married Child Accur. **0.9334**				With Single Child Accur. **0.8731**			
		0-25	25-50	50-75	75-100	0-25	25-50	50-75	75-100
Children	0-25	72.7	81.0	84.0	88.8	37.5	39.7	39.6	40.0
	25-50	61.8	77.0	81.5	87.7	45.7	49.3	51.8	60.3
	50-75	39.7	72.0	78.6	86.7	49.8	55.1	59.1	74.0
	75-100	12.6	45.3	66.8	83.8	57.7	64.3	72.0	84.3

If we instead used the original data, then the predictions change slightly[8] with the accuracy for single children being quite lower (0.7927).

Table 16: Parameter estimates

Param.	Estim.	Param.	Estim.	Param.	Estim.	Param.	Estim.
$\overline{c}^m$	-1032.83	γ	1.71	η^n	-0.42	η^s	4.79
ρ^n	6.18	ρ^s	0.09	λ^n	0.25	λ^s	0.17
α^{nm}	0.09	α^{nh}	0.81	θ^n	1.18	θ^s	0.50
α^{sm}	3.05	α^{sh}	0.45	χ^n	0.04	χ^s	64.19

The parameter estimates reported in Table 16 are quite interesting. First of all, we see that zero effort when the child is married induces a very low probability of living alone while the opposite occurs when the child is single. As one would expect, effort is a lot more costly for married children than for single children, reflecting perhaps the involvement of a spouse. It turns out that the mother does not like to live with her married child but that she does indeed like to live with her single child. We also see that the economies of scale are very different for single and for married children, showing increasing returns in total household income for married children and decreasing returns for single children. The effort cost is bigger for the mothers than for children in the single sample, while the opposite it is true for married children. This could reflect the fact that single children have different life-styles that married children, being more independent or more reluctant to share their privacy with others. Finally, we see that the share of total household consumption devoted to the mothers is bigger in the married children households than in the single ones.

[8] The estimation results with the original data are reported in Table 19 in Appendix D.

Figures 4 and 5 show the estimates and the data. Again we see how good are the estimates, and we also see that they look a bit better for married children. All in all, we think that this marital status model with sixteen parameters and thirty two observations has a very good fit with the data.

10 Predictions for 1990 of the Model with marital status

Like in Section 5 we use the estimated model with the 1970 income data to obtain new equilibria results and to compare them with observations in 1990 data. We use the same measure of accuracy of predictions as in Section 5. In order to deflate incomes in 1990 we also take the same CPI adjustment: 2.55 1990 dollars per a 1970 dollar.

The model predicts for singles a 70.8% of the actual increase in the data while for married children the increase is 39.9%. However during this period there was an increase in the number of singles, a factor that by itself predicts (meaning ignoring the role of income) a reduction of the fraction of elderly mothers living alone which is 59.8%. The overall prediction of mothers living alone is then 68.7%, that is, one half of the actual change in the data. If instead of the overall prediction we use the accounting statistics defined above we get 58.9%.

Table 17: Predictions of the Model with marital status for 1990
Percentage of mothers living alone

Error: **0.02627**	Mothers							
	With Married Child Error: **0.04400**				With Single Child Error: **0.01049**			
	81.7	85.7	87.8	90.3	40.2	41.1	43.2	43.6
Children	77.2	83.3	86.1	89.5	54.1	59.5	64.6	74.0
	65.2	79.1	83.6	88.4	63.4	72.2	78.2	84.5
	12.9	53.5	74.0	85.2	77.2	82.6	85.3	88.4

The properties of the model's predictions are reported in detail in Table 17. We see that the model replicates the uniform increase in all income groups. However, the model misses the tremendous increase in living alone percentage of poor mothers with a married rich child that has happened in the data where it went from 15.2% to 62.5%.

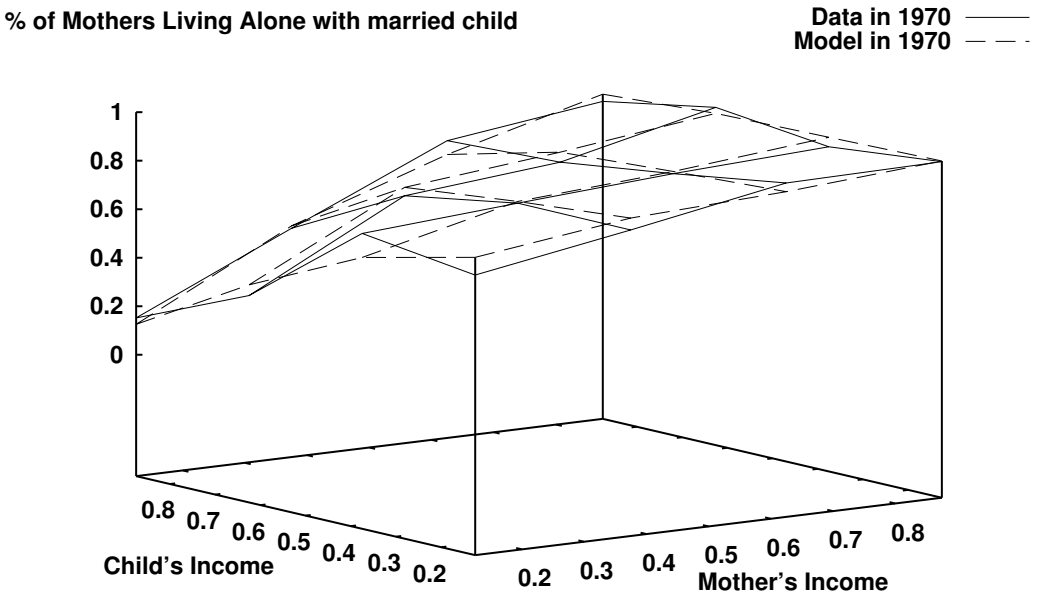

Figure 4: **Fraction of mothers living alone who have a married child in the model and in the 1970 data.**

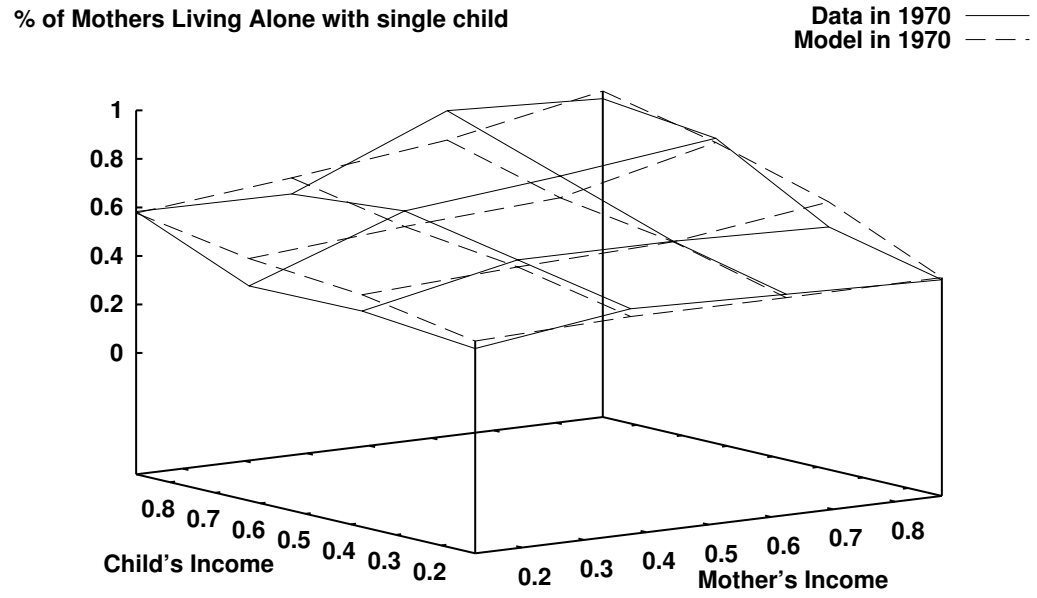

Figure 5: **Fractions of mothers living alone who have a single child in the model and in the 1970 data.**

While the model with marital status has a better fit with the data, it implies that changes in income account for a slightly lower fraction of the actual increase in the number of widows living alone: 74.4% in the model without marital status and 50.4% in the one with it.

11 Conclusion

We have documented the increase in the fraction of elderly widows that live alone and its relation both to their income and to their children's income. We have done so by using different data sets and making the assumption of stability of the intertemporal persistence of incomes across generations in order to be able to link mothers and children that live apart.

We have posed various versions of a model of the determination of the living arrangements based on making effort in controlling the outcome and where the income of the two parties plays a central role. We have estimated those models using 1970 data obtaining quite a good fit and replicating the strongly nonlinear patterns of the data.

We then used the models to make predictions about the prevailing living arrangements of 1990 based only on the incomes of mothers and children and we found that changes in income account for three quarters of the change in living arrangements of elderly widows between 1970 and 1990.

We have explored how different types of income changes have affected the living arrangements and we have found that the increase in the income of the widows accounts for two thirds of the total increase in the fraction of widows living alone; the increase in the income of children by itself reduces the fraction of widows living alone and the combined effect is what accounts for the rest up to the three quarters predicted by the model, reflecting the highly non-linear relation between incomes and living arrangements. From a different point of view, we have found that the change in relative income between mothers and children accounts for about one half the changes in living arrangements in the data while the increase in the levels of income accounts for about one quarter of the changes. From yet another type of decomposition of the changes in incomes, we have found that the change in average levels of income accounts for almost all of the increase in the fraction of widows living alone, while the change in the dispersion of incomes by itself reduces the fraction of widows living alone. Again, the combination of changes in the averages and in the dispersions of income is larger

than the sum of their individual effects.

We then explored whether various other characteristics of mothers and children in addition to their incomes play an important role in shaping living arrangements. We concluded that while some do not seem very important, or at least, not in obvious ways (number and sex of the children, and to some extent age of the children), marital status of the children seems to play an important role in determining whether elderly widows live with their children.

Consequently we then posed a model of the determination of living arrangements that incorporates the marital status of the children as well as the other characteristics that we found mattered in the simpler models. We went on estimating this model using 1970 data and we found that the model is very good in accounting for the patterns in data for both the mothers who have married children and the mothers who have single children. This extended model predicts that changes in incomes account for about one half of the actual changes in living arrangements, a lower number than the models that abstract from marital status.

All this leads us to conclude that the increase in income of the mothers, compounded by the general increase in income for the whole population, has been the most important factor in shaping the changes in living arrangements and that alternative explanations have played a much smaller role.

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Appendices

A Data Analysis

We use data from the 1970 and 1990 IPUMS, and from the 1993 AHEAD. The IPUMS database is a large (about two million individuals in 1970 and two and one half millions in 1990) representative sample of the Census and hence of all the US population, but is not a panel. Its main disadvantage is the impossibility to establish links between individuals from different households. So if an elderly widow is not living with her children we do not have any information about them. Fortunately, the 1993 AHEAD (Asset and Health Dynamics Among the Oldest Old), which is designed to collect data about elderly people, has this information. Several sections of the questionnaire are designed to get information about the children whether they are living in the elder's household or not. We construct tables of the joint distribution of income of the widows, their offspring and their living arrangements by merging the information from the AHEAD with the information from IPUMS. We explain in detail how we do it in Appendix B. We start describing how we choose our sample of elderly widows.

Age: Between 1970 and 1990 life expectancy of 65 year old women went from 17 to 19 years.[9] This was accompanied by a reduction in the disability rates among elderly people.[10]. To account for the increase in life span, we pose a slight difference in the definitions of elderly widows for the two periods. While in 1970 we select widows from 65 to 82 years, for 1990 we choose those widows from 67 to 84 years of age. The change in the age group that we look at has also the additional advantage of keeping nearly constant the fraction of widows (49.1% and 47.8% in 1970 and 1990 respectively) since the increase in life expectancy also affects men.

Number of children: We select elderly widows who gave birth to at least two children. In the IPUMS samples, the average number of children for elderly widows were 3.84 in 1970, and 3.64 in 1990. For the restricted sample of those elderly widows who gave birth to two

[9]See The Berkeley Mortality Database webpage: `http://www.demog.berkeley.edu/wilmoth/mortality.`
[10]See Manton, Corder, and Stallard (1997)

or more children, the average is 4.41 in 1970 and 4.14 in 1990, a difference of less than 10%. This small change in family size and the fact that there seems to be a weak relation between family size and living arrangement are the reasons why we abstract from family size in our model.

Living arrangements: There are four types of living arrangements that can be used to characterize the data, living alone, with children, with others and in an institution. An elderly widow is defined as living in an institution (or group quarters) if she lives with five or more individuals who are unrelated to the household head. This is the strategy suggested by Ruggles and Sobek (1995) in order to make definitions consistent over the 1970 and the 1990 census.

Table 18: Distribution of Widows by Living Arrangements in Percentages

Living Arrangement	1970	1990	Difference
Alone	52.1	64.2	12.1
With Children	32.0	21.0	-11.0
With Others	10.6	10.3	-0.3
In an Institution	5.3	4.5	-0.8

Table 18 shows the distribution of living arrangements of widows. Living with others and living in an institution are infrequent events and they have remained relatively constant. Hence, we abstract from those two living arrangements and we only consider the options of living alone or living with children. In 1970 62% of the widows that were not living with others or in institutions lived alone while in 1990 this fraction was 75.3%. The set of women that we look at constitute 66.6% of the unmarried women of age 65+ and 80% of the unmarried women for the age range defined previously.

B Living arrangements and income: Imputation Process

As we said above, the IPUMS data do not allow us to link the widows living alone and their children. However, the 1993 AHEAD does.[11] We make the assumption that the intertemporal persistence of relative income is the same in 1970 as it was in 1990 and 1993. We construct the 1970 and 1990 mother-children pairs with the following detailed steps.

1. **1993 AHEAD**. We analyze only children and widow-mothers pairs in 1993 AHEAD. If the widow is living alone and has more than one child, we randomize and select one of the children. We define four equal size income groups for both widows and children, and we calculate the joint distribution of incomes of mothers and children. That is:

		Mother				
		Poor	Less Poor	Less Rich	Rich	Marginal
Child	Poor	P_{11}	P_{12}	P_{13}	P_{14}	25.0
	Less Poor	P_{21}	P_{22}	P_{23}	P_{24}	25.0
	Less Rich	P_{31}	P_{32}	P_{33}	P_{34}	25.0
	Rich	P_{41}	P_{42}	P_{43}	P_{44}	25.0
	Marginal	25.0	25.0	25.0	25.0	100.0

   Where $P_{i,j}$ is the proportion of mothers with type j income with i type children.

2. **Subsample of individuals living alone.** From the 1970 and 1990 IPUMS, we select a subsample of children living alone of the same size than the widows living alone.

3. **Subsample of individuals living together.** We select from the IPUMS the pairs of mothers and children that live together.

4. **Marginal distribution of incomes of mothers and children: 1970 and 1990 IPUMS.** We then sort all the children and all the widows into four income groups of equal size. We obtain the fractions of those that live together and denote them $T^t_{i,j}$ for t equal to 1970 and 1990.

[11] The AHEAD data only surveys individuals older than 70 years. Fortunately, the samples seem to be consistent (the percentage of 70 and older widows living alone in the 1990 IPUMS is very close to the 1993 AHEAD, 75.6% and 72.1% respectively). Since we are only using from AHEAD the intergenerational distribution of income, we can use AHEAD.

5. **Imputation of the joint distribution of incomes.** Using the $P_{i,j}$ from the AHEAD we obtain

$$A^t_{i,j} = P_{i,j} - T^t_{i,j} \tag{15}$$

where $A^t_{i,j}$ are the fraction of elderly widows with income i and with children with income j who are living alone in year t.

In other words, we randomly choose from the IPUMS a sample of children and mothers living alone and generate the child-mother income pairs according to the 1993 AHEAD joint distribution. This allows us to get the average income for mothers and children in each pair.

C Living arrangements, income and marital status: Imputation Process

Now we are interested in constructing mothers-children pairs attending to their incomes and the children's marital status. For that, we assume that the intergenerational persistence of income and children's marital status is the same in 1970 and 1990 as it was in 1993.

We start calculating the 1993 AHEAD joint distribution of income and marital status, $P^z_{i,j}$ (with $\sum_{i,j,z} P^z_{i,j} = 1$). So, we control the joint income distribution we got in Appendix B by the children's marital status. Then, we turn to the IPUMS and we also control de marginal distributions of income by the marital status of the children.

Next, we get the fractions of those that live together in each group and denote them $T^z_{i,j}$. However, because of the marginal distributions by marital status are different ones across years, we can not impute the joint income distribution to the census sample as before. We solve this imputing in one dimension: the children marginal distribution or the mothers marginal distribution. For instance, if we impute the marginal of the children:

$$\hat{P}^{z,t}_{i,j} = P^{z,t}_{i,\cdot} \frac{(P^z_{i,j})}{(P^z_{i,\cdot})}, \quad i,j = 1,...,4 \quad z = n,s \quad t = 70,90 \tag{16}$$

Note that the marginal distribution of the children is the same:

$$\hat{P}^{z,t}_{i,\cdot} = \sum_j \hat{P}^{z,t}_{i,j} = P^{z,t}_{i,\cdot} \quad i,j = 1,...,4 \quad z = n,s \tag{17}$$

However, the marginal distribution of the mothers after the imputation could be different to the original one in the data. When higher is the difference, the higher the error of the imputation.[12]

So we select the marginal distribution to impute of these of mothers' and children's that it produce less error, and this was the marginal distribution of the children.

Thus, we get the joint income distribution, $\hat{P}_{i,j}^{z,t}$, and we calculate the shares of individuals living alone in each group, $A_{i,j}^{z,t}$, as[13]:

$$A_{i,j}^{z,t} = \hat{P}_{i,j}^{z,t} - T_{i,j}^{z,t} \tag{18}$$

So we generate the child-mother income pairs of these who are living alone according to the imputed 1993 AHEAD joint distribution, and finally we get the average income for mothers and children in each pair.

D Estimation results in 1970 with real data

Table 19: Predictions of the Model with marital status for 1970
Percentage of mothers living alone

Accur. **0.9089**	Mothers With Married Child Accur. **0.9333**				Mothers With Single Child Accur. **0.7927**			
	72.8	81.1	84.0	88.8	37.0	38.2	38.8	39.6
Children	61.8	77.1	81.5	87.7	44.5	47.9	50.3	58.3
	39.4	72.1	78.7	86.7	48.2	53.1	56.7	70.9
	12.6	45.0	66.8	83.8	55.0	60.9	68.0	83.0

[12] Note that the error is zero if the resulting marginal distribution is the same as the one in the 1993 AHEAD sample. This was the case in the Appendix B when we imputed the joint income distribution.

[13] Like we are imputing not the joint distribution but the marginal one, it exists the possibility that $T_{i,j}^{z,t}$ (from the data) would be bigger than $\hat{P}_{i,j}^{z,t}$. In this case $A_{i,j}^{z,t}$ it would have a negative sign which it has no sense. Fortunately, we did not find this problem in our samples.